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The Hall effect and magnetoresistance of a two-dimensional electron gas upon scattering on microinhomogeneities of a magnetic field

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Abstract. The conductivity of a two-dimensional degenerate electron (2DEG) gas has been calculated in a microscopically inhomogeneous magnetic field of Abrikosov's vortices. The mean free path of electrons is assumed to be much larger than the vortex diameter 2λ (in this case the vortices play the role of additional asymmetric scatterers). It is shown that the Hall constant for a *degenerate* electron gas contains the Hall factor defined by the value of parameter $k_F\lambda$ (k_F is the Fermi wavevector of the electron). The Hall resistance is determined by the mean value of magnetic field in the 2DEG plane in the $k_F\lambda \gg 1$ limit only. The case of a multiquantum vortex has also been investigated (the number of flux quanta in the vortex is $\gamma \gg 1$). In the case of classical small-angle scattering ($k_F\lambda \gg \gamma$) the Hall resistance is determined by the mean value of the magnetic field. The Hall resistance decreases monotonically if scattering is of the classical, but not small-angle, type and is saturated in the region of quantum scattering. It is shown that the presence of vortices leads to finite resistance (even in the absence of other scatterers). The transport scattering time of electrons on vortices is calculated.

1. Introduction

The present work is devoted to the calculation of the conductivity of a two-dimensional degenerate electron gas (2DEG) with a large mean free path in a microscopically inhomogeneous magnetic field of Abrikosov's vortices. Such a system may be obtained [1] if a heterostructure with a type II superconductor film sputtered on its surface is placed into an external homogeneous magnetic field (figure 1). The external magnetic field is split into separate quanta inside and in the vicinity of the superconductor. The characteristic diameter of the vortex is 2λ, where $\lambda \simeq 0.1\ \mu$m is the penetration depth. Thus, 2D electrons move in the field of chaotically distributed Abrikosov's vortices† (figure 2). By varying the external field H we may vary the vortex concentration N. This case is considered when the mean free path l_i of electrons is much larger than the vortex diameter. Hence, the electron is affected by the magnetic field only over a small part of

† The fact that the vortex system forms either a regular or an irregular lattice depends on the pinning conditions in the system. We shall consider the case of the irregular vortex lattice as was the case in [1]. The case of the regular vortex lattice is qualitatively different from that under consideration and is not discussed below.

To avoid misunderstanding, let us emphasize that the 2D electrons in question are in the normal state.

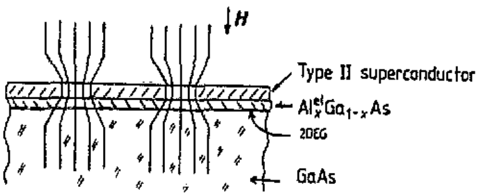

Figure 1. The system considered.

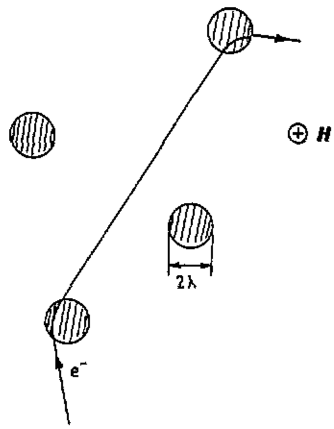

Figure 2. The trajectory of the 2D electron in the field of chaotically distributed Abrikosov's vortices. The electron mean free path is much larger than the vortex diameter 2λ.

its trajectory. As a result, the vortices play the role of additional asymmetric electron scatterers. Note that the vortex is an essential quantum scatterer since the classical angle of the deviation of an electron passing through the region of the vortex magnetic field is always of the same order of magnitude as the angle of quantum mechanical diffraction. This is valid even when the electron wavelength is much smaller than the vortex diameter: $\lambda_F \ll \lambda$. This is because only a single flux quantum is enclosed within the vortex. Therefore, the scattering should be described by quantum mechanics.

The main problem is to calculate the Hall resistance in such a system when magnetic field is strongly inhomogeneous. Will the Hall resistance be determined by the mean value of the magnetic field in the system? Also of interest is the value of the classical magnetoresistance in such a system.

First let us briefly report on the results obtained and offer a qualitative explanation of them. The main result resides in the fact that, if the mean free path is much larger than the vortex size, the value of the Hall resistance for *degenerate* electron gas is

$$\rho_{xy} = (H/nec)\alpha \tag{1}$$

where H is the mean value of magnetic field in the 2DEG plane which coincides with the value of external homogeneous magnetic field applied to the structure. The value of the Hall factor α is determined by parameter $k_F\lambda$, where k_F is the Fermi wavevector of the electron. The parameter $k_F\lambda$ defines the degree of quantum scattering of the electron on the vortex. The degree of quantum scattering increases with decreasing $k_F\lambda$. Later it is assumed that the inequality $\lambda_F, \lambda \ll l_i, N^{-1/2}$ is fulfilled. Here the relationship between λ_F and λ and between l_i and $N^{-1/2}$ may be arbitrary ($N^{-1/2}$ is the vortex spacing). It is shown that the Hall resistance is determined by the mean value of the magnetic field (i.e. $\alpha = 1$) only in the limit $k_F\lambda \gg 1$. In this case the electron scattering on the vortex is the small-angle (characteristic scattering angle $\theta_0 = 2/k_F\lambda \ll 1$). Therefore, in the pure system ($l_i \to \infty$) the electron trajectory consists of a large (about $2\pi/\theta_0$) number of segments about $1/2\lambda N$ long which form an almost closed curve approaching (at $k_F\lambda \to \infty$) a circle with a radius defined by the mean magnetic field in the plane. As $k_F\lambda$ decreases, the Hall resistance decreases monotonically, tending to zero in the limit $k_F\lambda \to 0$. As is

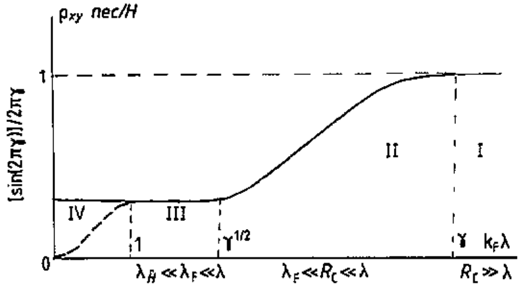

Figure 3. Dependence of the Hall resistance on parameter $k_F\lambda$ for the case when each vortex contains many flux quanta γ ($\gg 1$). See section 3.

known [2], the transverse force acting on the electron from the vortex is equal to zero in the quantum limit $\lambda/\lambda_F \to 0$, if the number γ of enclosed flux quanta is equal to an integer or half-integer ($\gamma = \Phi/\Phi_0$, $\Phi_0 = 2\pi\hbar c/|e|$). In the case of the Abrikosov vortex, $\gamma = \frac{1}{2}$.

In order to understand better the behaviour of the Hall resistance in systems with strongly inhomogeneous distribution of magnetic field, we have also considered the case of the multiquantum vortex ($\gamma \gg 1$). The result is represented in figure 3. The Hall factor is equal to unity only in the limit of classical small-angle scattering ($k_F\lambda \gg \gamma$). In this parameter region the cyclotron radius R_c in the internal vortex field is large compared with the vortex diameter and the characteristic scattering angle $\lambda/R_c \sim \gamma/k_F\lambda$ is small. The reason why the Hall factor in this limit is equal to unity may be understood from the following simple considerations. The value of the Hall force that appears in the field term of the Boltzmann kinetic equation is defined by

$$N \int d^2X\, \Psi^* \hat{F}_y \Psi \tag{2}$$

where Ψ is the wavefunction of the scattering electron, $\hat{F}_y = -(e/c)\hat{V}_x\tilde{H}$ the operator of the Lorentz force acting on the electron and $\tilde{H}$ the internal magnetic field in the vortex. The domain of integration in (2) is the vortex area. In the classical small-angle scattering limit in question, equation (2) is reduced to $-N(e/c)V_F \int d^2X\, \tilde{H} = -(e/c)V_F H$ since for small-angle scattering $V_x \simeq V_F$ (V_F is the Fermi velocity), and $N\int d^2X\, \tilde{H} = H$ from the condition of magnetic flux conservation. Hence, in this limit the Hall factor $\alpha = 1$. From this it is clear that the Hall resistance is maximum upon classical small-angle scattering and decreases with increasing characteristic scattering angle. Indeed, consider the parameter region $\gamma^{1/2} \ll k_F\lambda \ll \gamma$ (see figure 3, region II). Here the cyclotron radius in the internal vortex field is small compared with the vortex diameter, but it is still large in comparison with the electron Fermi wavelength. In this parameter region the scattering is not of a small-angle nature since the electron penetrates into the vortex by a small depth of the order of the cyclotron radius only. The Hall resistance decreases monotonically in proportion to the value $(R_c/\lambda)^2 \propto (k_F\lambda/\gamma)^2$ in accordance with the small part of the vortex area which affects the electron upon its scattering. The decrease in the Hall resistance with decreasing $k_F\lambda$ can be easily understood from the following considerations. As is known [2], in the limiting quantum case $k_F\lambda \ll 1$ (region IV in figure 3) the transverse force acting from the vortex is determined by the phase difference $2\pi\gamma$ gained by the quasiparticle along the trajectories on the right and left of the vortex, the value of the force being periodically dependent on the number of enclosed flux quanta. As a result, in the quantum limit the vortex behaves as if the number of enclosed quanta were $\gamma - [\gamma]$, where $[\gamma]$ is the integral part of γ. By contrast, in the classical limit ($k_F\lambda \gg \gamma$) the Hall force is proportional to the number of enclosed quanta. It should be noted again that, in the case in question $\gamma \gg 1$, the change from the 'classical' value $\alpha = 1$ to the quantum value $\alpha = [\sin(2\pi\gamma)]/2\pi\gamma$ occurs in the region of *classical non-small-angle scattering.*

It is shown that in the absence of other scatterers the presence of vortices leads to finite resistance. The resistivity caused by scattering on vortices is

$$\rho_{xx} = m/ne^2\tau_{tr} \qquad 1/\tau_{tr} = \tfrac{1}{8}\theta_0^2(1/\tau_0) \qquad 1/\tau_0 = 2\lambda N V_F \propto H \tag{3}$$

where τ_{tr} is the transport scattering time of electrons on vortices, $1/\tau_0$ is the electron–vortex collision frequency†. Therefore, in the *degenerate* electron gas the classical magnetoresistance is not equal to zero and is proportional to the modulus of H. Such behaviour of the magnetoresistance is due to the linear dependence of scatterer concentration, i.e. Abrikosov's vortices, on H.

2. Single-quantum vortex ($\gamma = \frac{1}{2}$)

In this section we calculate the Hall resistance and magnetoresistance of 2DEG in the field of chaotically distributed Abrikosov's vortices. The electron distribution function f_p is determined from the Boltzmann kinetic equation

$$eE \cdot v \frac{\partial f_0}{\partial \varepsilon} = \sum_q K_{p+q\to p} f_{p+q} - K_{p\to p+q} f_p) - \frac{f_p - f_0}{\tau_i}. \tag{4}$$

Here the first term on the right-hand side of the kinetic equation (let us denote it by $St^v\{f_p\}$ describes collisions with vortices whilst the second term describes collisions with impurities. It should be emphasized that the presence of vortices is taken into consideration only in the collision term. Also note that transition probability K is an asymmetric value. Proceeding from the fact that the scattering on vortices is elastic, we may seek the solution of equation (4) in the form $f_p = f_0(\varepsilon_p) + (\partial f_0/\partial\varepsilon)(p \cdot C)$, where f_0 is the equilibrium Fermi function and C is so far an arbitrary vector. Then the electron–vortex collision integral is transformed to

$$St^v\{f_p\} = \frac{\partial f_0}{\partial \varepsilon} \sum_q K_q(q \cdot C) = \frac{\partial f_0}{\partial \varepsilon} N v_F \sum_q |F_q|^2 (q \cdot C) \tag{5}$$

where F_q is the scattering amplitude on the vortex corresponding to the transferred momentum q. Since $q = -(p \times h)\sin\theta - p(1 - \cos\theta)$, where θ is the scattering angle and h is the unit vector along the magnetic field in the vortex (the normal to the 2D plane). Finally we obtain that the electron–vortex collision integral can be separated into two terms

$$St^v\{f_p\} = -(\alpha e/c)(v \times H)(\partial f_p/\partial p) - (f_p - f_0)/\tau_{tr}. \tag{6}$$

When obtaining the first term in (6), we used the relationship between the vortex

† Here the real vortex ($\gamma = \frac{1}{2}$) and the case of small-angle scattering ($k_F\lambda \gg 1$) are implied. The definition of angle θ_0 is given above.

concentration and mean magnetic field in the plane $N(\Phi_0/2) = H$. The factor α is determined by the expression

$$\alpha = +\frac{k_F}{\pi}\int_{-\pi}^{+\pi} d\theta \sin\theta\,|F(\theta)|^2 \qquad F(\theta) \equiv F_q. \tag{7}$$

The transport scattering time of electrons on vortices is determined by

$$\tau_{tr}^{-1} = N v_F \int_{-\pi}^{+\pi} d\theta\,(1-\cos\theta)|F(\theta)|^2. \tag{8}$$

As a result, in the absence of other scatterers the presence of vortices leads to finite resistance in the system.

Thus, from (6) it follows that the problem is reduced to that of electron movement in a homogeneous field αH. Hence, for the value of the Hall resistance, we obtain (1). Here it should be noted that the small-angle scattering region where the solution does not have the form of the sum of the incident and diverging cylindrical waves may contribute significantly to the transverse (Hall) force acting on the electron from the vortex. This is the case, for instance, in the quantum limit $k_F\lambda \to 0$ when the long-range action of the vortex field gives rise to a transverse force of a specific diffraction origin which is not expressed in terms of the scattering amplitude [2]. Therefore, calculations should be carried out with care and the contribution of the force of the diffraction origin (the so-called Iordanskii force [3, 4]) should be estimated.

The scattering amplitude should be known in order to calculate (7) and (8). In this section we consider the limiting case $k_F\lambda \gg 1$ when the scattering is of the small-angle type (the scattering angle is of the order of $\theta_0 = 2/k_F\lambda \ll 1$). This enables the eikonal approximation to be used for calculating the scattering amplitude [5]. The wavefunction behind the vortex is obtained from the incident wavefunction by integration of the xth component of the vector potential along unperturbed straight trajectories (x axis):

$$\Psi_f(r) = \exp(ikx)\exp\left(i\frac{e}{\hbar c}\int_{-\infty}^{x} A_x(r)\,dx\right) \qquad r = x, y. \tag{9}$$

Let us choose the vector potential $A(r)$ in the form $A_r = 0$, $A_\theta(r) = \tilde{H}r/2$, at $r \leqslant \lambda$ and $\Phi_0/4\pi r$ at $r \geqslant \lambda$, where $\tilde{H}$ is the vortex internal field. Here we use the model of the vortex with a sharp boundary and constant internal field. It will be shown below that the result for the Hall constant in the limit $k_F\lambda \gg 1$ is independent of the model of field distribution in the vortex.

Equation (9) is valid at $x \ll k_F\lambda^2$. To calculate the scattering amplitude, it is sufficient to know the wavefunction at distances x such that $\lambda \ll x \ll k_F\lambda^2$. Later it will be shown that, when calculating the scattering amplitude, the characteristic values of $|y|$ are approximately equal to λ. In this case $|y| \ll x$ and the integral in the exponent in (9) may be extended to infinity:

$$\begin{aligned}&\Psi_f(r) = \exp(ik_F x)\exp[i\varphi(y)]\\ &\varphi(y) = [\pi/2 - \tan^{-1}(\lambda^2/y^2 - 1)^{1/2}]\,\mathrm{sgn}(y) + (y/\lambda)(1 - y^2/\lambda^2)^{1/2}.\end{aligned} \tag{10}$$

In equation (10) $(1-y^2/\lambda^2)^{1/2}$ at $|y|>\lambda$ should be substituted for zero. The scattering amplitude is calculated by means of (10) [5]:

$$F(\theta)=\left(\frac{k_F}{2\pi i}\right)^{1/2}\int_{-\infty}^{+\infty}\mathrm{d}y\,\exp(-ik_F y\theta)\{\exp[i(\varphi(y))]-1\}. \tag{11}$$

The characteristic scattering angle in (11) is $\theta_0 \ll 1$. The asymptotic behaviour of the scattering amplitude (11) is as follows: at $|\theta|\ll\theta_0$, $|F(\theta)|^2\propto 1/\theta^2$; at $\theta_0\ll|\theta|\ll 1$, $|F(\theta)|^2\propto 1/|\theta|^5$ ((integral (11) is calculated by introducing the factor $\exp(-\delta|y|)$ with subsequent passage to the limit $\delta\to 0$, $\delta>0$ [5]). Note that at $|\theta|\ll\theta_0$ the asymptotic behaviour coincides with the angular dependence of the small-angle scattering amplitude on an infinitely thin magnetic string [6], which is natural. The power asymptotic behaviour at $|\theta|\gg\theta_0$ is due to the use of the model of the vortex with a sharp edge.

Now let us turn to calculation of (7) and (8). The calculations presented in the appendix yield

$$\alpha=\frac{|e|}{\pi\hbar c}\int_{-\infty}^{+\infty}\mathrm{d}y\int_{-\infty}^{+\infty}\mathrm{d}x\,\tilde{H}(x,y)=\frac{|e|}{\pi\hbar c}\frac{\Phi_0}{2}=1. \tag{12}$$

For $1/\tau_{tr}$ we obtain equation (3). Thus in the limit $k_F\lambda\gg 1$ the Hall resistance is determined by the mean value of magnetic field. Note that α is proportional to the integral of the internal magnetic field in the vortex over its area, i.e. to the flux in the vortex. Hence, the value of (12) is independent of the model of field distribution in the vortex.

Finally, let us make two more important points. As is shown in the appendix,

$$1/\tau_{tr}=Nv_F\int_{-\infty}^{+\infty}\mathrm{d}y\,\theta_{cl}^2(y)/2$$

where $\theta_{cl}(y)$ is the scattering angle of electron for the impact parameter y calculated by the formulae of classical mechanics at $k_F\lambda\gg 1$. Therefore in this limit the transport cross section is determined by its classical expression. Let us note that the differential scattering cross section therewith is appreciably different from the classical value. The property indicated does not depend on the vortex model in use and is common to all the cases when scattering may be described by the eikonal approximation. For instance, in [7] it was shown that in the case of conventional potential scattering the transport cross section coincides with its classical value even in the limit of Born small-angle scattering provided that the electron wavelength is small compared with the size of the scatterer.

The second point concerns the calculation of α. The calculation presented in the appendix corresponds to the integral (7) at $\theta\to 0$ taken in the sense of the principal value (at $\theta\to 0$, $\alpha\propto\int \mathrm{d}\theta/\theta$). As mentioned above, the calculation of the transverse force in the region of very small angles is incorrect when carried out with the use of the scattering amplitude. Let us verify the obtained value of $\alpha=1$, i.e. that in the limit $k_F\lambda\gg 1$ the contribution of the diffraction region is actually small. At the same time let us give another way of calculating α. The transverse force may be calculated by immediate use of the solution of (9). The identity expressing the law of momentum conservation has the form [4]

$$\oint_{\mathscr{L}}\left[\frac{1}{2m}\left(i\hbar\frac{\partial\Psi^*}{\partial x_k}-\frac{e}{c}A_k\Psi^*\right)\left(-i\hbar\frac{\partial\Psi}{\partial x_j}-\frac{e}{c}A_j\Psi\right)+\mathrm{cc}-\frac{\hbar^2}{4m}\frac{\partial}{\partial x_k}\frac{\partial}{\partial x_j}(\Psi^*\Psi)\right]n_k\,\mathrm{d}l$$

$$\equiv\oint_{\mathscr{L}}G_{kj}n_k\,\mathrm{d}l=\int_{S(\mathscr{L})}\mathrm{d}^2x\,\Psi^*\hat{F}_j\Psi \tag{13}$$

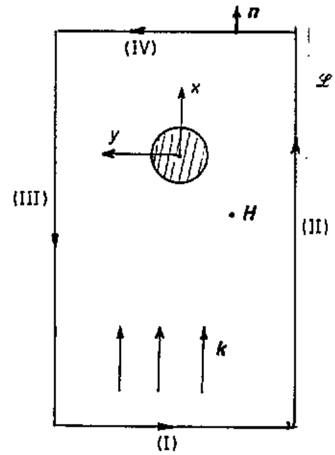

Figure 4. Path $\mathscr{L}$ of integration for calculation of the transverse force acting on the electron from the vortex (see (13)).

where the term on the left-hand side produces momentum carried over by scattered electrons and the term on the right yields the mean value of the force acting on the electrons; $S(\mathscr{L})$ is the area limited by the contour $\mathscr{L}$, and n_k the vector of the normal to contour $\mathscr{L}$. Choose contour $\mathscr{L}$ (figure 4) so that it encloses the vortex. Then for $j = y$ using (9) it can easily be shown that the integral $\int G_{kj} n_k \, \mathrm{d}l$ along sides I, II and III of the contour is equal to zero and the integral along side IV is

$$(|e|/c)v_{\mathrm{F}} \int_{(\mathrm{IV})} \mathrm{d}y \int_{-\infty}^{x} \mathrm{d}x_1 \, \tilde{H}(x_1, y) = (|e|/c)v_{\mathrm{F}}(\Phi_0/2).$$

After multiplying this value by the vortex concentration N, we obtain for (2) the value $(|e|/c)V_{\mathrm{F}}H$, which corresponds to $\alpha = 1$. As $k_{\mathrm{F}}\lambda$ decreases, the Hall resistance decreases monotonically, reducing to zero at $k_{\mathrm{F}}\lambda \to 0$. When $k_{\mathrm{F}}\lambda \ll 1$, $\alpha \propto (k_{\mathrm{F}}\lambda)^2$.

In an experiment [1] it was actually observed that the Hall resistance of a 2DEG in the magnetic field of Abrikosov's vortices coincides with the Hall resistance in a homogeneous magnetic field (to an accuracy of less than 1%). In the experiment the value of $k_{\mathrm{F}}\lambda$ was about 29, and the mean free path $l_i \doteq 2\,\mu$m. The experimental points lie on the same straight line well in fields less than and greater than 200 G (in a field $B \simeq 200$ G the vortices were strongly overlapped so that the variation in magnetic field in the 2DEG plane does not exceed several per cent).

The linear dependence (versus H) of magnetoresistance (see (3)) was also experimentally observed in the region of magnetic fields until the vortices were not overlapping. The experiment [1] corresponded to the case of a sufficiently high electron concentration that the electron wavelength was small compared with the vortex diameter. It would be interesting to verify experimentally the predicted effect of deviation of the Hall resistance from the value defined by the mean value of magnetic field in the system. This could be done by measuring the Hall resistance on heterostructures with one and the same superconducting film but different electron concentrations n (each time the electron mean free path should be large compared with the vortex diameter). If the Hall resistance (multiplied by nec) is plotted as a function of external magnetic field, the following pattern should be observed. In the region of magnetic fields that are much smaller than the characteristic field of vortex overlapping the straight lines corresponding to various electron concentrations will have different slopes which are smaller for lower electron concentrations. This is due to the difference in the Hall factor. In the region of magnetic fields larger than the field of vortex overlapping (when the magnetic field in the system is uniform) the straight lines corresponding to different electron concentrations should coincide.

3. Multiquantum vortex ($\gamma \gg 1$)

In this section we consider the case when many quanta of flux γ are enclosed inside the vortex. The aim is to study the dependence of the Hall factor α on the parameter $k_F\lambda$. It is still assumed that the vortex diameter is small compared with the mean free path and the model of the vortex with a sharp edge and constant internal field is used. The result is shown in figure 3. In region I ($k_F\lambda \gg \gamma$) the cyclotron radius R_c in the vortex internal field is large compared with the vortex diameter 2λ. Hence, the scattering is of a small-angle nature ($\theta \simeq \lambda/R_c \ll 1$). Therefore, in this parameter region the electron wavefunction Ψ may be found by the method described in section 2 (integrating the vector potential along straight trajectories). As a result, for Ψ we obtain an expression of type (9). Then using equation (13) and choosing again contour $\mathcal{L}$, as shown in figure 4, we obtain $(|e|/c)V_FH$ for (2) and, hence, $\alpha = 1$ (as previously H is the mean magnetic field in the plane). Now let us consider region II (see figure 3) where $\gamma^{1/2} \ll k_F\lambda \ll \gamma$ which may be written as $\lambda_F \ll R_c \ll \lambda$. The first inequality implies that scattering both inside and outside the vortex may be described by the quasiclassical method whilst the second inequality implies that scattering is not of a small-angle type. In order to calculate the transverse force, let us use equation (13) and choose the vortex perimeter as the integration path. Since the electron movement in the whole space may be described by classical mechanics, the tensor of momentum flux density may be written as $G_{kj} = v_k p_j$, v_k and p_j being the kinematic velocity and momentum. Since the electron penetrates into the vortex to a small depth of about R_c, it is specularly reflected from the vortex boundary (the local angle of incidence is equal to the angle of reflection) in the zero approximation with respect to the parameter R_c/λ. For this reason, the value $\oint_{\mathcal{L}} G_{ky}n_k\,\mathrm{d}l \equiv \lambda \int \mathrm{d}\theta\, v_r p_y$ is equal to zero as electrons with impact parameters $(+y)$ and $(-y)$ make an opposite contribution to this value (v_r is the projection of the electron velocity on the radius vector; the incident electron flux is directed along the x axis). It is obvious that the difference between the angle of incidence and the angle of reflection is of the order of $R_c/\lambda \ll 1$. This value will define the degree of scattering asymmetry on the vortex in this parameter region. Thus we obtain

$$\oint G_{ky}n_k\,\mathrm{d}l \propto \lambda v_F p_F (R_c/\lambda) \propto v_F p_F R_c$$

(only the parametric dependence and not the numerical coefficient which is about unity is of interest). Multiplying the obtained value by the vortex concentration and using the relationship $N\gamma\Phi_0 = H$ between the concentration and mean magnetic field, for (2) we finally obtain $(|e|/c)v_F H(R_c/\lambda)^2$ and hence

$$\alpha \simeq (R_c/\lambda)^2 \simeq (k_F\lambda/\gamma)^2. \tag{14}$$

Thus, in this parameter region the Hall resistance decreases monotonically (figure 3). Estimating the value of α at point $k_F\lambda \simeq \gamma^{1/2}$, where the region of applicability of equation (14) terminates, we obtain $\alpha \simeq 1/\gamma$. Now refer to region IV (figure 3). Using the results obtained in [2] for the scattering of a plane electron wave on an infinitely thin magnetic string, for $\lambda \to 0$ we have

$$\alpha = [\sin(2\pi\gamma)]/2\pi\gamma. \tag{15}$$

Clearly, in the region $k_F\lambda \leqslant 1$, α is also equal to (15) to within an order of magnitude (if γ is not equal to an integer or a half-integer so that $\sin(2\pi\gamma) \simeq 1$). This case is shown by the full curve in region IV (figure 3). However, if γ is an integer or a half-integer, then, at $\lambda \to 0$, α reduces to zero. However, on the boundary of the quantum region ($k_F\lambda \simeq 1$), α should also reach about $1/\gamma$ (since in the region $k_F\lambda > 1$, when the situation outside the vortex is classical, there can be no difference between the cases when γ is exactly equal to an integer or a half-integer or is different from them). The case of an integer or half-integer γ is shown by the broken curve in region IV (figure 3). Therefore, on the boundaries of region III the values of α have the same orders of magnitude and there is no parametric variation in α. In region III the magnetic length in the internal vortex field is smaller than the electron wavelength $\lambda_{\tilde{H}} \ll \lambda_F$ (or $\hbar\tilde{\omega}_c \gg \varepsilon_F$, $\tilde{\omega}_c$ being the cyclotron frequency in the internal vortex field). The electron movement outside the vortex may be described classically; yet in the internal region the electron state is strongly quantized. The electron penetrates into the vortex during scattering to a small depth of the order of the magnetic length. Unfortunately, because of mathematical problems we cannot determine the form of the dependence of α in this region but there are qualitative physical considerations that suggest the absence of a parametric variation in α.

A brief account of this work has been published in [8].

Acknowledgments

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Appendix

Let us calculate α in (7). Since $|F(\theta)|^2 \propto 1/|\theta|^5$ at $|\theta| \gg \theta_0$, (7) is determined by the integration region $|\theta| \simeq \theta_0 \ll 1$. Then the integration for θ in (7) may be extended to infinity and, with the aid of (11), α at $\theta \neq 0$ may be written as

$$\alpha = \frac{1}{2\pi^2}\int_{-\infty}^{+\infty} \xi\,d\xi \int_{-\infty}^{+\infty} d\rho\,\exp(-\delta|\rho|)\exp(i\varphi(\rho))\exp(-i\rho\xi)\int_{-\infty}^{+\infty} d\rho_1\exp[-i\varphi(\rho_1)]$$
$$\times\exp(i\rho_1\xi)\exp(-\delta|\rho_1|) = \frac{1}{2\pi^2}\int_{-\infty}^{+\infty} d\xi\int_{-\infty}^{+\infty} d\rho\,\exp(-\delta|\rho|)\exp[i\varphi(\rho)]$$
$$\times\exp(-i\rho\xi)\int_{-\infty}^{+\infty} d\rho_1\exp(-\delta|\rho_1|)\exp[-i\varphi(\rho_1)]\left(-i\frac{\partial}{\partial\rho_1}\right)\exp(i\rho_1\xi). \tag{A1}$$

In integral (A1) we have introduced the new integration variables $y/\lambda = \rho$, $k_F\lambda\theta = \xi$ ($\delta \to 0$, $\delta > 0$). Following integration for ρ_1 by parts in (A1), integration with respect to ξ yields $2\pi\delta(\rho_1 - \rho)$. Finally we obtain

$$\alpha = \frac{1}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}\rho \frac{\mathrm{d}\varphi(\rho)}{\mathrm{d}\rho} = \frac{1}{\pi}[\varphi(+\infty) - \varphi(-\infty)] = 1. \tag{A2}$$

Note that using the relationship

$$\frac{\partial\varphi(y)}{\partial y} = \frac{e}{\hbar c}\int_{-\infty}^{+\infty} \mathrm{d}x \frac{\partial A_x}{\partial y} = \frac{e}{\hbar c}\int_{-\infty}^{+\infty} \mathrm{d}x \left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\right) = -\frac{e}{\hbar c}\int_{-\infty}^{+\infty} \mathrm{d}x\, \tilde{H}(x, y) \tag{A3}$$

where $\tilde{H}$ is the internal field in the vortex, (A2) may be also written as

$$\alpha = -\frac{e}{\pi\hbar c}\int_{-\infty}^{+\infty} \mathrm{d}y \int_{-\infty}^{+\infty} \mathrm{d}x\, \tilde{H}(x, y) = \frac{k_{\mathrm{F}}}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}y\, \theta_{\mathrm{cl}}(y) \tag{A4}$$

where $\theta_{\mathrm{cl}}(y)$ is the angle of electron scattering on the vortex for an impact parameter y calculated from the formulae of classical mechanics for $k_{\mathrm{F}}\lambda \gg 1$. Thus, in this limit, α coincides with the value calculated with the help of classical mechanics (and equal to unity). This is due to the small-angle character of scattering.

Now let us turn to calculation of (8). Using the small value of θ and introducing again the variables ρ and ξ we may write (8) as

$$\frac{1}{\tau_{\mathrm{tr}}} = \frac{\Omega}{4\pi}\int_{-\infty}^{+\infty} \mathrm{d}\xi \int_{-\infty}^{+\infty} \mathrm{d}\rho\, \exp(-\delta|\rho|)\{\exp[\mathrm{i}\varphi(\rho)] - 1\}\frac{\partial}{\partial\rho}\exp(-\mathrm{i}\xi\rho)$$
$$\times \int_{-\infty}^{+\infty} \mathrm{d}\rho_1\, \exp(-\delta|\rho_1|)\{\exp[-\mathrm{i}\varphi(\rho_1)] - 1\}\frac{\partial}{\partial\rho_1}\exp(\mathrm{i}\xi\rho_1) \tag{A5}$$

$$\Omega = \frac{N v_{\mathrm{F}}}{k_{\mathrm{F}}^2\lambda}.$$

Following integration by parts for ρ and ρ_1 in (A5), integration with respect to ξ yields $2\pi\delta(\rho_1 - \rho)$. Finally we obtain

$$\frac{1}{\tau_{\mathrm{tr}}} = \frac{\Omega}{2}\int_{-\infty}^{+\infty} \mathrm{d}\rho \left(\frac{\mathrm{d}\varphi}{\mathrm{d}\rho}\right)^2 = 2\Omega\int_{-1}^{+1} \mathrm{d}\rho\,(1 - \rho^2) = \tfrac{1}{6}\theta_0^2\frac{1}{\tau_0}. \tag{A6}$$

Using (A3), we may also write (A6) as

$$\frac{1}{\tau_{\mathrm{tr}}} = \frac{N v_{\mathrm{F}}}{2k_{\mathrm{F}}^2}\left(\frac{e}{\hbar c}\right)^2 \int_{-\infty}^{+\infty} \mathrm{d}y \left(\int_{-\infty}^{+\infty} \mathrm{d}x\, \tilde{H}(x, y)\right)^2 = \frac{N v_{\mathrm{F}}}{2}\int_{-\infty}^{+\infty} \mathrm{d}y\, \theta_{\mathrm{cl}}^2(y). \tag{A7}$$

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